

Ph.D Entrance Test 2021

पेपर/विषय का नाम Name of the Paper/Subject	MATHEMATICS	पेपर/विषय का कोड Paper/Subject Code	111321
रोल नं. Roll No.	अभ्यर्थी का नाम Name of Candidate		
केन्द्र का नाम Name of the Centre	अभ्यर्थी के हस्ताक्षर Signature of Candidate		

क्र. सं./Serial No.

समय: 2:00 घंटा

अधिकतम अंक: 100

Time: 2: 00 Hours

Maximum Marks: 100

अभ्यर्थियों के लिए अनुदेश

- बुकलेट में ओएमआर शीट और दो सील हैं। अभ्यर्थी सबसे पहले ओएमआर शीट प्राप्त करने के लिए बुकलेट के सबसे ऊपर की सील हटाकर निकालें। दूसरी सील परीक्षा शुरू होने के दो मिनट पहले हटाई जाएगी।
- परीक्षा शुरू करने से पहले अभ्यर्थी प्रश्नपत्र पुस्तिका और ओएमआर उत्तर-पत्रक पर अपना रोलनं. लिखना और निर्धारित स्थानों पर हस्ताक्षर करना सुनिश्चित करें।
- इस प्रश्नपत्र पुस्तिका में इस कवर पृष्ठ के अलावा कुल 100 प्रश्न हैं। रफ कार्य करने के लिए प्रश्न पत्र के अन्त में उपलब्ध खाली पृष्ठों का प्रयोग करें।
- प्रत्येक प्रश्न के लिए चार वैकल्पिक उत्तर (a), (b), (c) और (d) दिए गए हैं। अभ्यर्थी जिस एक उत्तर को सही समझता है, उसका चयन करने के बाद उत्तर-पत्रक में गोले को अंकित करे/रंगे।
- गोले को रंगने के लिए काले/नीले बॉल पेन का प्रयोग करें।
- निम्नलिखित उदाहरण देखें।
उदाहरण
1. 20 और 12 का जोड़ होता है
(a) 32 (b) 38 (c) 31 (d) 34
उपयुक्त प्रश्न का सही उत्तर (a) है, जिसे ओएमआर उत्तर-पत्रक में निम्नलिखित रूप में अंकित करें:

1	<input checked="" type="radio"/>	b	c	d
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- आधा रंगा हुआ, हल्के रूप से अंकित, गोले में सही या गलत के निशान को ऑप्टिकल स्कैनर द्वारा इसे गलत उत्तर के रूप में पढ़ा जाएगा और इसे गलत माना जाएगा।
- परीक्षा कक्ष छोड़ने से पहले ओएमआर उत्तर पुस्तिका निरीक्षक को अवश्य सौंप दें।
- ओएमआर उत्तर पत्र को सीधे रखें। इसे मोड़ें आदि नहीं।
- सभी प्रश्न अनिवार्य हैं, प्रत्येक प्रश्न एक अंक का है।
- कैलकुलेटर/मोबाइल/कोई भी इलेक्ट्रॉनिक मद/आपत्तिजनक सामग्री की घटना को अनुचित प्रयोग के साधन का मामला माना जायेगा।

INSTRUCTIONS TO THE CANDIDATES

- The booklet contains OMR sheet and having two seals. Candidates will first open the booklet by removing the seal at the top to get the OMR sheet. Second seal will be removed two minutes before the commencement of the examination.
- Before starting the Examination, the candidate must write her/his Roll Number in the Question Booklet and the OMR Answer Sheet; in addition to putting signature at the places provided for the purpose.
- This Question Booklet consists of this cover page, and a total 100 items. Use Blank pages available at the end of Question Booklet for rough work.
- There are four alternative answers to each item marked as (a), (b), (c) and (d). The candidate will have to select one of the answers that is considered to be correct by her/him. S/he will mark the answer considered to be correct by filling the circle.
- Use black/blue ball point pen to darken the circle.
- See the following illustrations.
Illustration:
1. The sum of 20 and 12 is
(a) 32 (b) 38 (c) 31 (d) 34
The Correct answer of item 1 is (a), which should be marked in OMR Answer Sheet as under:

1	<input checked="" type="radio"/>	b	c	d
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- Half filled, faintly darkened, ticked or crossed circles will be read as wrong answers by the optical scanner and will be marked as incorrect.
- The OMR Answer Sheet must be handed over to the invigilator by the candidate before leaving the Examination Hall.
- Keep OMR Sheet straight. Do not fold it.
- All questions are compulsory, each question carries one mark.
- Incidence of carrying of calculator/mobile/any electronic device/objectionable material will be treated as unfair means case.

परीक्षा नियंत्रक

Controller of Examination

कृपया नोट करें कि अर्थ विभेद/दुविधा की स्थिति में अंग्रेजी में छपे प्रश्न को अंतिम माना जाएगा।

Please note that in case of any confusion, the question printed in English will be considered final.

Mathematics
Section A

1. The additive group $\text{Hom}_{\mathbb{Z}}(\mathbb{Z}/2\mathbb{Z}, \mathbb{Z}) =$
 - a) \mathbb{Z}
 - b) 0
 - c) $\mathbb{Z}/2\mathbb{Z}$
 - d) $2\mathbb{Z}$

2. Let $f(z) = u + iv$ be an entire function with the property that $u_x v_y - u_y v_x = 1$ in \mathbb{C} . Then $f(z) =$
 - a) $az + b$, where a and b are constants with $|b| = 1$
 - b) $az + b$, where a and b are constants with $|a| = 1$
 - c) $az + b$, where a and b are constants with $|b| = 2$
 - d) $az + b$, where a and b are constants with $|a| = 2$

3. Which of the following function has essential singularity at $z = 0$
 - a) $f(z) = \frac{\sin(z^2)}{z^2(z-a)}$
 - b) $f(z) = \frac{\sin(z)}{z(z-a)}$
 - c) $f(z) = e^{1/z^2}$
 - d) $\frac{1}{z^2(z-a)}$

4. Let p be a prime integer. Then number of finite groups, up to isomorphism, of order p^2 is
 - a) 1
 - b) 2
 - c) 3
 - d) 4

5. Which of the following is a class equation of the quaternion group Q_8
 - a) $2+2+2+2$
 - b) $1+1+2+2+2$
 - c) $1+1+1+1+2+2$
 - d) $1+1+1+1+1+1+2$

6. Consider the following statements:
P: A Hermitian matrix is positive semi-definite if and only if the determinants of all its principal sub-matrices are non-negative.
Q: Any positive semi-definite matrix has a unique square root.
Which of the above statements hold true?
 - a) Only P
 - b) Only Q
 - c) Both P and Q
 - d) Neither P nor Q

7. The kernel of the homomorphism $\phi : \mathbb{C}[x, y] \rightarrow \mathbb{C}[t]$ defined by $\phi(x) = t^2$ and $\phi(y) = t^3$ is the ideal
 - a) $(y^3 - x^2)$
 - b) $(y^3 - x^3)$
 - c) $(y^2 - x^2)$
 - d) $(y^2 - x^3)$

16. The characteristic of the partial differential equation $2u_x + 3u_y = u$ are
- Exponential curves along which u remains constant
 - Straight lines along which u varies exponentially with x
 - Straight lines along which u remains constant
 - Exponential curves along which u varies linearly with x

17. Initial value problem,

$$\left| \frac{dy}{dx} \right| + |y| = 0 \quad \text{with} \quad y(\pi) = 0,$$

has

- Unique solution
 - No solution
 - Exactly two solutions
 - Infinite solutions
18. On which of the given intervals will the following initial value problem have a unique, continuous, solution?

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} e^{2t} & \frac{t}{t-2} \\ te^t & t \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} \tan t \\ t + t^2 \end{bmatrix}, \quad \begin{bmatrix} x(3) \\ y(3) \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix},$$

- $-2 < t < 2$
 - $\pi/2 < t < 3$
 - $2 < t < 3\pi/2$
 - $\pi/2 < t < 3\pi/2$
19. Consider the nonhomogeneous equation

$$y'' + 6y' + 9y = (2t + t^4)e^{-3t}.$$

By the method of undetermined coefficients, there is a solution to the equation which is of the form $y(t) = P(t)e^{-3t}$ where $P(t)$ is a polynomial. The degree of $P(t)$ is

- 2
 - 3
 - 5
 - 6
20. Pick the region in which the following differential equation is elliptic

$$xu_{xx} + 2u_{xy} + yu_{yy} - 3u_x + 7u_y = 11$$

- $xy > 1$
- $xy < 1$
- $xy = 1$
- $xy = 1/2$

21. Let there be a Cauchy problem

$$\frac{\partial^3 u}{\partial x^3} - \frac{\partial u}{\partial y} = 0, \quad u(x, 0) = 2x\sqrt{x}, \quad 0 \leq x \leq 1.$$

Then its solution is given by

- a) $2x\sqrt{x}[1 - \frac{27}{2}y]$ b) $2x\sqrt{x}[1 - \frac{27}{4}y]$
c) $2x\sqrt{x}[1 + \frac{27}{2}y]$ d) $2x\sqrt{x}[1 + \frac{27}{4}y]$

22. Let n be a non-negative integer. Then the eigenvalues of the Sturm-Liouville problem

$$y'' + \lambda y = 0,$$

with boundary conditions $y(0) = y(2\pi)$, $y'(0) = y'(2\pi)$ are

- a) n b) $n^2\pi^2$
c) $n\pi$ d) n^2

23. If $y' - x \neq 0$, a solution of the differential equation

$$y'(y' + y) = x(x + y)$$

is given by

- a) $y = 1 - x - e^{-x}$ b) $1 - x + e^x$
c) $1 + x + e^{-x}$ d) $1 + x + e^x$

24. Let there be an initial value problem $u_t + (u^2/2)_x = 0$ with data

$$u(x, 0) = \begin{cases} 1, & \text{if } x \leq 0 \\ 0, & \text{if } x > 0. \end{cases}$$

Then solution is given by

- a) $u(x, t) = \begin{cases} 1, & \text{if } x \geq t/2 \\ 0, & \text{if } x < t/2. \end{cases}$ b) $u(x, t) = \begin{cases} 1, & \text{if } x \leq t/2 \\ 0, & \text{if } x > t/2. \end{cases}$
c) $u(x, t) = 0$ d) Solution does not exist

25. Let $u(x, t)$ be the solution of $u_{tt} - u_{xx} = 0$, $0 < x < 1$, $t > 0$, $u(x, 0) = x(1 - x)$, $u_t(x, 0) = 0$. Then $u(\frac{1}{4}, \frac{1}{2})$ is

- a) 0 b) $-1/16$
c) $3/16$ d) $1/16$

26. Suppose $1 < m < n$ and let $G = \langle (12, \dots, m), (12, \dots, n) \rangle$. Then the group G contains

- a) a 2-cycle b) a 3-cycle
c) a 4-cycle d) a 5-cycle

43. In maximization LPP, if a variable corresponding to positive $z_j - c_j$ is entered and leaving variable rule is properly followed, then
- the next basic solution will not be BFS
 - the solution may be bounded or unbounded
 - the value of the objective function will decrease
 - the value of the objective function will increase
44. If $x = \xi$ is an isolated zero of $f(x)$ with multiplicity $m \neq 1$, then the rate of convergence of Newton's method is
- Linear
 - Quadratic
 - Linear if $m = 2$
 - Quadratic if $m = 2$
45. Given $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, the maximum possible basic solution is
- 3
 - 4
 - 2
 - 6
46. If $P(A \cap B) = \frac{1}{2}$, $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$, $P(A) = p$, $P(B) = 2p$, then the value of p is given by
- 1/3
 - 7/18
 - 4/9
 - 1/2
47. An unbiased coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then n is equal to
- 10
 - 12
 - 14
 - 16
48. If the mean of a binomial distribution is 25, then its standard deviation lies in the interval given below
- (0, 25]
 - [0, 25)
 - (0, 5]
 - [0, 5)
49. If in a moderately skewed distribution the value of mode and mean are 6λ and 9λ respectively, then the value of the median is
- 8λ
 - 7λ
 - 6λ
 - 5λ
50. If a, b, c, d are constants such that a and c are of opposite signs and r is the correlation coefficient between X and Y , then the correlation coefficient between $aX + b$ and $cY + d$ is equal to
- $\frac{a}{c} r$
 - $\frac{c}{a} r$
 - r
 - $-r$

Section B
Research Methodology

51. Which of the following is a feature of Quantitative research
- a) Approach to enquiry is structured
 - b) Sample size is small
 - c) Approach to enquiry is unstructured
 - d) Emphasis on description of variables
52. The missing number in the sequence is
0, 6, 24, 60, 120,, 336, is
- a) 240
 - b) 220
 - c) 280
 - d) 210
53. Under which of the following categories SyLOW's theorems fall
- a) Descriptive Research
 - b) Sample Survey
 - c) Fundamental Research
 - d) Applied Research
54. The next step after the formulation of the hypothesis is
- a) Data collection
 - b) Statement of objectives
 - c) Analysis of data
 - d) Selection of research tools
55. The binary relation "is a brother of" is
- a) Symmetrical
 - b) Transitive
 - c) Reflexive
 - d) Anti-symmetrical
56. Deductive logic studies the way in which a premise may
- a) Support and entail a conclusion
 - b) Not support but entail a conclusion
 - c) Neither support nor entail a conclusion
 - d) Support a conclusion without entailing it
57. Which of the following is the initial step of a research
- a) Searching a problem
 - b). Selecting a problem
 - c) Finding a problem
 - d) Identifying a problem
58. Bibliography given in a research report:
- a) Helps those interested in further research
 - b) Shows vast knowledge of the researcher
 - c) Has no relevance to research
 - d) Has no connection to the report

59. Which of the following is a Type-1 Error
- a) Acceptance of a null hypothesis when it is false
 - b) Rejection of a null hypothesis when it is true
 - c) Rejection of the hypothesis when it is true
 - d) None of these
60. Which of the following is a Type-2 Error
- a) Acceptance of a null hypothesis when it is false
 - b) Rejection of a null hypothesis when it is true
 - c) Rejection of the hypothesis when it is true
 - d) None of these
61. The primary aim of selecting a hypothesis is
- a) To widen the scope of a research study
 - b) To bring the usefulness of a research study
 - c) To bring focus to a research study
 - d) To bring importance of a research study
62. Which of the following is another name for null hypothesis
- a) Hypothesis of difference
 - b) Hypothesis of no difference
 - c) Hypothesis of association
 - d) Hypothesis of point-prevalence
63. Hypotheses are usually not formulated if the aim of a study
- a) To test an assertion by way of causality
 - b) To test an assertion by way of association
 - c) To explore where very little is known
 - d) To validate the prevalence of something or establish its existence
64. Which of the following is not a programming language?
- a) JAVA
 - b) FORTRAN
 - c) COBOL
 - d) UBUNTU
65. A research design serves the function
- a) To detail the procedures for making study important
 - b) To detail the procedures for undertaking a study
 - c) To detail the procedures for making the study useful
 - d) All of these
66. Which of the following are the problems with the data from secondary sources
- a) Personal biasness
 - b) Validity
 - c) Reliability
 - d) All of these

67. A hyperboloid of one sheet contains
- a) One parameter family of straight lines
 - b) Two parameter family of straight lines
 - c) Three parameter family of straight lines
 - d) Four parameter family of straight lines
68. Primary sources are those where
- a) Government collects the data
 - b) Some private agency collects the data
 - c) You collect the information from respondents
 - d) None of these
69. Which of the following variables cannot be expressed in quantitative terms?
- a) Intelligence quotient (IQ)
 - b) Economic status
 - c) Numerical aptitude
 - d) Emotional quotient (EQ)
70. Which of the following is a negation of 'all students are intelligent'
- a) No student is intelligent
 - b) All students are not intelligent
 - c) Some students are intelligent
 - d) There exists a student who is not intelligent
71. 1 TB (terabyte) equals
- a) 1024 GB
 - b) 1000 GB
 - c) 1023 GB
 - d) 1022 GB
72. Computer program that translates computer code written in one programming language into target language is called
- a) Operating system
 - b) Compiler
 - c) Main program
 - d) None of these
73. The term research ethics is concerned with
- a) Using prescribed format of a thesis
 - b) Evidence based research
 - c) Quantitative methods
 - d) Qualitative methods
74. Which of the following branches of mathematics doesn't use abstract methods for its study
- a) Homological algebra
 - b) Category theory
 - c) Linear algebra
 - d) Numerical analysis

75. Which of the following is usually not considered a part of Analysis
- a) Fourier analysis
 - b) Differential Geometry
 - c) Category Theory
 - d) Numerical analysis
76. The mathematician Alexander Grothendieck is known for his work in
- a) Fourier analysis
 - b) Differential Geometry
 - c) Noncommutative Algebra
 - d) Algebraic Geometry
77. Which of the following number is usually associated with Indian mathematician Ramanujan (Taxicab number)
- a) 1729
 - b) 1730
 - c) 1731
 - d) 1733
78. The number of integers between 100 and 1000 that are divisible by 7 is equal to
- a) 127
 - b) 129
 - c) 128
 - d) 133
79. The statement "there are infinitely many pythagorean triplets" is a statement in
- a) Fourier analysis
 - b) Analytical Geometry
 - c) Number Theory
 - d) Real Analysis
80. Which of the following is not considered the basis of scientific method?
- a) Empirical evidence
 - b) Opinions and responses of people
 - c) Laboratory experimentations
 - d) Objective considerations
81. Which of the following is essential for starting a research project
- a) Good hypothesis
 - b) Motivation
 - c) Passion
 - d) A well formulated research problem
82. Mathematical research primarily make use of
- a) Deductive reasoning
 - b) Inductive reasoning
 - c) Laboratory experimentations
 - d) Objective considerations
83. Research is always about
- a) Verifying the old knowledge
 - b) Exploring new knowledge
 - c) Filling the gap between knowledge
 - d) All of these
84. A person writes all the numbers from 0 to 99. The number of times digit 5 will be written is
- a) 18
 - b) 19
 - c) 20
 - d) 21

85. What is equivalent of the statement 'All cars are jeeps' ?
- a) All non-cars are non-jeeps
 - b) All non-jeeps are non-cars
 - c) All jeeps are cars
 - d) none of these
86. Which of the following is a general framework for undertaking a research
- a) Research design
 - b) Research synopsis
 - c) Research hypothesis
 - d) none of these
87. Which of the following mathematician has proved Fermat's Last Theorem
- a) Fermat
 - b) Andrew Wiles
 - c) Richard Taylor
 - d) Ramanujan
88. The process of making and recording observations is called
- a) Data Analysis
 - b) Data Measurement
 - c) Data Collection
 - d) Data Processing
89. Prior to undertaking a research project the process of critical reading of the available literature on the topic is called
- a) Reviewing the literature
 - b) Researching of literature
 - c) Assessment of literature
 - d) Analysing of literature
90. A fundamental characteristic of a good research is:
- a) Generality
 - b) Usefulness
 - c) Objectivity
 - d) Comprehensibility
91. When the population is heterogeneous which of the following sampling method is preferred
- a) Random sampling
 - b) Cluster sampling
 - c) Quota sampling
 - d) Stratified sampling
92. When the population is difficult to enumerate or is hidden which of the following sampling techniques is preferred
- a) Random sampling
 - b) Snowball sampling
 - c) Quota sampling
 - d) Stratified sampling
93. Two events are mutually exclusive if
- a) Both can occur at the same time
 - b) Occurrence of one implies the occurrence of the other
 - c) Occurrence of one implies the non-occurrence of the other
 - d) none of these

94. Which of the following is a habit or a trait of a critical mind
- a) Desire to follow evidence and reason wherever they may lead
 - b) Desire of follow the established rules
 - c) Desire of follow the authority
 - d) All of these
95. Which of the following is not a charateristic of a research
- a) Research is rigorous
 - b) Research is systematic
 - c) Research need not be verifiable
 - d) Rsearch needs to be objective
96. In research we use methods that
- a) have been tested for validity
 - b) have been tested for reliability
 - c) are unbiased
 - d) are subjective and simple
97. The main objective of a qualitative study is to
- a) describe the variation in a phenomenon
 - b) describe the deversity in a situation or attitude
 - c) describe the variation and deversity in a phenomenon
 - d) all of these
98. Number theory uses methods of
- a) Analysis
 - b) Geometry
 - c) Algebra
 - d) all of these
99. A null hypothesis is
- a) when there is a difference between variables
 - b) no difference between certain characteristics of a population
 - c) same as the research hypothesis
 - d) always subjective
100. Historical research falls into the category of
- a) Fundamental research
 - b) Experimental research
 - c) Descriptive research
 - d) None of these

SPACE FOR ROUGH WORK